Name of College - S.S College, J. Bad
Deptt - Mathematics
Topic - Pedal Equation (Tangent and Normal diff. calculus)
Class - B.Sc I (Hons)
Date - 20-07-2020
Time - 10:15 to 11:00 and 11:00 to 11:45
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Study material →
Since equation of curve is either cartesian equation or polar equation

But when the equation of any curve is given in terms of (p, r) where p = length of perpendicular from the pole on the tangent and r is the radius vector

then this form is called pedal form.

Method to transform the equation of curve in polar form

(1) To find the pedal equation of a curve from its polar form

STEP I → Write down the polar equation of curve

$$f(r, \theta) = 0 \qquad \text{——— (I)}$$

Step II → If p be the perpendicular distance of tangent from pole drawn at any point (r, θ) and ϕ be the angle between radius vector and tangent at (r, θ)

then

$$\tan\phi = r \cdot \frac{d\theta}{dr} \qquad \text{——— (II)}$$

and

$$p = r\sin\phi \qquad \text{——— (III)}$$

Now eliminate ϕ from (I) (II) and (III)

and then we get an relation equation in terms of p and r

and this is required pedal equation of curve.

in pedal form

Step I - Write down equation of curve
$f(x,y) = 0$ ——— (1)

Step II - Find the equation of tangent at any point (x_1, y_1)

using $\frac{y-y_1}{x-x_1} = \left(\frac{dy}{dx}\right)_{x_1,y_1}$ $\Rightarrow y-y_1 = (x-x_1)\left(\frac{dy}{dx}\right)_{x_1,y_1}$

$\Rightarrow x\left(\frac{dy}{dx}\right)_{x_1,y_1} - y - x_1\frac{dy}{dx} + y_1 = 0$

$\Rightarrow x\left(\frac{dy}{dx}\right)_{(x_1,y_1)} - y - \left(x_1\frac{dy}{dx} - y_1\right) = 0$

If p is the perpendicular distance from origin to tangent

then $p = \frac{x_1\left(\frac{dy}{dx}\right)_{x_1,y_1} - y_1}{\sqrt{1+\left(\frac{dy}{dx}\right)^2_{x_1,y_1}}}$ ——— (II)

Also $r^2 = x^2 + y^2$ ——— (III)

On eliminating x and y from (I) (II) and (III), we shall get an equation in terms of p and r and this will be the required pedal equation of the curve.

Problem Based on Pedal Equation

Type I → When the equation of curve is $f(x,y) = 0$

Problem 1

1. Show that the pedal equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with respect to its centre is

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} - \frac{r^2}{a^2b^2}$$

Solution :→ By the question,

$$f(x,y) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0 \quad \text{(i)}$$

equation of tangent at (x_1, y_1) is given by

$$\frac{x x_1}{a^2} + \frac{y y_1}{b^2} - 1 = 0 \quad \text{(ii)}$$

If p be the perpendicular distance from origin

$$p = \frac{1}{\sqrt{\frac{x_1^2}{a^4} + \frac{y_1^2}{b^4}}}$$

$$\Rightarrow \frac{1}{p} = \sqrt{\frac{x_1^2}{a^4} + \frac{y_1^2}{b^4}}$$

$$\Rightarrow \frac{1}{p^2} = \frac{x_1^2}{a^4} + \frac{y_1^2}{b^4} \quad \text{(iii)}$$

Also $x_1^2 + y_1^2 = r^2 \quad \text{(iv)}$

On Elimination x_1, y_1 from (i) (iii) and (iv)

$$\begin{vmatrix} \frac{1}{a^2} & \frac{1}{b^2} & 1 \\ \frac{1}{a^4} & \frac{1}{b^4} & \frac{1}{p^2} \\ 1 & 1 & r^2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a^2 & b^2 & p^2 \\ 1 & 1 & 1 \\ a^4 & b^4 & r^2p^2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a^2 & b^2 & p^2 \\ a^4 & b^4 & r^2p^2 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow a^2 \begin{vmatrix} b^4 & r^2p^2 \\ 1 & 1 \end{vmatrix} - b^2 \begin{vmatrix} a^4 & r^2p^2 \\ 1 & 1 \end{vmatrix} + p^2 \begin{vmatrix} a^4 & b^4 \\ 1 & 1 \end{vmatrix}$$

$$\Rightarrow a^2(b^4 - r^2p^2) - b^2(a^4 - r^2p^2) + p^2(a^4 - b^4) = 0$$

$$\Rightarrow a^2b^4 - a^2r^2p^2 - b^2a^4 + b^2r^2p^2 + p^2a^4 - p^2b^4 = 0$$

$\Rightarrow a^2b^4 - b^2a^4 = a^2r^2p^2 - b^2r^2p + p^2b^4 - p^2a^4 = 0$

$\Rightarrow a^2b^2(b^2 - a^2) = p^2[a^2r^2 - b^2r^2 + b^4 - a^4] = 0$

$\Rightarrow a^2b^2(b^2-a^2) = p^2[r^2(a^2-b^2) + (b^2-a^2)(b^2+a^2)]$

$\Rightarrow a^2b^2(b^2-a^2) = p^2[(b^2-a^2)(b^2+a^2) - r^2(b^2-a^2)]$

$\Rightarrow a^2b^2 = p^2[a^2+b^2-r^2]$

$\Rightarrow \frac{1}{p^2} = \frac{a^2+b^2-r^2}{a^2b^2}$

$\Rightarrow \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2} - \frac{r^2}{a^2b^2}$ which is the required pedal equation of the given curve.

Problem Find the pedal equation of the parabola $y^2 = 4ax$ with respect to its focus.

Solution → Here $f(x,y) \equiv y^2 - 4ax = 0$

Since focus is $(a,0)$

By transfering the origin to the focus $(a,0)$ equation of parabola becomes

$y^2 = 4a(x+a)$

$\Rightarrow y^2 = 4ax + 4a^2$

The equation of tangent to the parabola at the point (x_1,y_1) is

$yy_1 = 2a(x+x_1) + 4a^2$

$\Rightarrow 2ax - yy_1 + 4a^2 + 2ax_1 = 0$

Now p = perpendicular distance from origin of tangent drawn at (x_1,y_1)

$p = \frac{4a^2 + 2ax_1}{\sqrt{4a^2 + y_1^2}} = \frac{4a^2 + 2ax_1}{\sqrt{4a^2 + 4ax_1 + 4a^2}}$

$\Rightarrow p = 4a^2 + 2ax_1$

[illegible]

$\Rightarrow p = \frac{[illegible]}{[illegible]}$

$p = \frac{[illegible]}{[illegible]} = [illegible]$

$\therefore p^2 = a^2 (2a + x_1)$ [illegible]

Also $r^2 = x_1^2 + y_1^2$

$= x_1^2 + 4ax_1 + 4a^2$

$= (x_1 + 2a)^2$

$\therefore r = x_1 + 2a$.

$p^2 = a(2a + x_1)$

$= a \cdot r$

$\therefore p^2 = ar$ is the required pedal equation